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MODEL OF A NONSTATIONARY ISOTROPIC TURBULENT FLOW OF A LIQUID WITH POLYMER ADDITIVES

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A closed equation is investigated for a function that describes the evolution in time of the energy distribution of turbulent velocity pulsations over different length scales in an isotropic flow of dilute solutions of linear high-molecular-weight polymers. A program for a numerical solution of this equation is developed. Results are obtained for the time variation of turbulent pulsation energy, dissipation rate, efficiency of polymer additives, and other characteristics for different Reynolds numbers and different concentrations of the polymer additives. The results obtained indicate decreased dissipation of the turbulent flow and are in good qualitative agreement with existing experimental facts.

Introduction. In [1] a closed equation was obtained for the function $P_t(r)$ that describes the energy distribution of turbulent velocity pulsations over different scales of length in an isotropic flow of dilute solutions of linear high-molecular-weight polymers. The influence of small polymer additions on the turbulent flow structure is taken into account by introducing the additional term $I(r, t)$ in the equation for $P_t(r)$. This term is completely determined in terms of the sought function $P_t(r)$ and parameters of the solution. In deriving an expression for $I(r, t)$ use was made of the concept of a mechanism of interaction of polymer molecules and turbulence, proposed in [2, 3]. This mechanism, in essence, reduces to the fact that the solution viscosity increases strongly at points of the flow where tensile deformation occurs.

In [4] the equation for the function $P_t(r)$ was solved in the stationary case. The stationary solution became possible upon introducing into the equation for $P_t(r)$ a term that models the pumping of turbulent energy. The solution for the stationary case made it possible to demonstrate rather easily the main qualitative effects of the interaction of polymer molecules and turbulence.

The aim of this work is to obtain a numerical solution and analyze the obtained results in a nonstationary case, i.e., to study the influence of polymer additives on the character of evolution of the turbulent flow structure of the liquid.

1. Closed Nonstationary Equation for the Function $P_t(r)$. The equation for the function $P_t(r)$ is given by formulas (11), (12)-(26) in [1]. Here we rewrite it in dimensionless form, dropping the term that describes energy pumping into the flow, since in the subsequent discussion we will solve a nonstationary problem of the structure evolution for a turbulent velocity pulsation field from a prescribed initial state, described by the function $P_0(r)$, without pumping of turbulent energy. The equation in dimensionless variables appears as

$$\frac{\partial P_t(\rho)}{\partial t'} = \frac{\partial}{\partial \rho}\left\{ \left[\frac{2}{\mathrm{Re}_0} + 2\gamma \int_0^{\rho} \sqrt{\rho P_t(\rho)}\, d\rho \right] \left(\frac{\partial}{\partial \rho} + \frac{4}{\rho} \right) P_t(\rho) - \frac{n}{\mathrm{Re}_0} \left[G(\rho,\ t) \frac{1}{\rho} P_t(\rho) \right] \right\}. \tag{1}$$

We write the initial and boundary conditions as

$$P_t(\rho)\Big|_{t=0} = 2\rho \exp(-\rho^2), \quad P_t(\rho)\Big|_{\rho=0} = 0, \quad P_t(\rho)\Big|_{\rho=\infty} = 0. \tag{2}$$

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In (1) and (2) use is made of the notation

$$\rho = \frac{r}{L_0}, \quad P_t(\rho) = \frac{L_0 P_t(r/L_0)}{\frac{2}{3} q(0)}, \tag{3}$$

$$t' = \frac{t\sqrt{\left(\frac{2}{3} q(0)\right)}}{L_0}, \quad \mathrm{Re}_0 = \frac{L_0 \sqrt{\left(\frac{2}{3} q(0)\right)}}{\nu}. \tag{4}$$

Here t denotes the dimensionless time and $n = \nu_s/\nu$ denotes the ratio of the viscosity factors of the solution at points of molecular tension and the solvent. This number is proportional to the concentration of polymer molecules C_p: $n \sim C_p N$, where N can be assumed approximately equal to the number of monomer links in a polymer molecule [2, 3]. In studying a polymer of one specific type at constant temperature we can consider the number n as the polymer additive concentration in water.

The function $G(\rho, t)$ is interpreted by the following formulas:

$$G(\rho, t) = 10^{-2} [I_0(\omega_1) I_1(\omega_2) + I_1(\omega_1) I_0(\omega_2)], \tag{5}$$

$$\omega_{(1,2)} = \omega_{(1,2)}(\rho, t) = 5/\tau^2 \mathrm{Re}_0^3 \varepsilon(t) (1 \pm Y(\rho, t)), \tag{6}$$

$$\tau = T/\tau_0, \quad \tau_0 = L_0^2/\nu. \tag{7}$$

Here T is the characteristic relaxation time for the polymer molecule, and τ_0 is the characteristic time of viscous dissipation on the length scale L_0.

The dissipation rate $\varepsilon(t)$ is determined by the formula

$$\varepsilon(t) = \frac{10}{\mathrm{Re}_0} \lim_{\rho \to 0} \frac{\partial}{\partial \rho} P_t(\rho). \tag{8}$$

The function $Y(\rho, t)$ is determined from the relation

$$Y(\rho, t) = \frac{\partial}{\partial \rho} P_t(\rho) \Big/ \frac{\partial}{\partial \rho} P_t(\rho)\Big|_{\rho=0}. \tag{9}$$

Expressions for $I_k(\omega_i)$, $k = 0, 1$, $i = 1, 2$ are given by formulas (24)-(26) in [1]. It is also noted there that the form of expression (6) for $\omega_{(1,2)}(\rho, t)$ is valid when a change in the number Re_0 is due to a change in the characteristic velocity $\sqrt{2/3 q(0)}$ of pulsations at a constant value of the length scale L_0.

We note that the value of the function $Y(\rho, t)$ at any values of the variables does not exceed unity. The quantity $\varepsilon(t)$ at any value of t is also limited by some maximum value that can be calculated by solving the equation for the case of no polymer additives. Therefore for the prescribed value of the parameter ε and for each value of Re we can calculate the function $G(\rho, t)$ for values of Y from the interval $[0-1]$ and for values of $\varepsilon(t)$ from the region of values of this quantity and then use it in solving Eq. (1), choosing values of this function that correspond to the quantities $Y(\rho, t)$ and $\varepsilon(t)$ at each specific point ρ, t. By choosing specific values of the quantities T, ν, L_0 ($T = 10^{-3}$ sec, $\nu = 10^{-6}$ m^2/sec, $L_0 = 10^{-3}$ m) and thus determining the value $\tau = 10^{-3}$ we can calculate the function $G(\rho, t)$ at different values of the number Re_0. The form of this function is shown in [1] for some values of Re_0.

2. Numerical Solution of the Equation for $P_t(\rho)$. To solve Eq. (1) numerically, it is convenient to introduce additional notation and perform a series of transformations.

We denote

$$F_t = \frac{2}{\mathrm{Re}_0} + 2\gamma \int\limits_0^\rho \sqrt{\rho P_t}\, d\rho\,, \quad S_t = \int\limits_0^\rho P_t\, d\rho\,, \tag{10}$$

where we assumed $P_t = T_t(\rho)$, and we write the initial equation (1) together with the initial and boundary conditions as a system of differential equations in partial derivatives

$$\frac{\partial P_t}{\partial t'} = \frac{\partial}{\partial \rho} \left\{ F_t \left(\frac{\partial P_t}{\partial \rho} + \frac{4P_t}{\rho} \right) - \frac{n}{\mathrm{Re}_0}\, G \frac{P_t}{\rho} \right\}, \tag{11}$$

$$\frac{\partial F_t}{\partial \rho} = 2\gamma \sqrt{\rho P_t}\,, \tag{12}$$

$$\frac{\partial S_t}{\partial \rho} = P_t \tag{13}$$

with the boundary conditions at $\rho = 0$

$$P_t \Big|_{\rho=0} = 0\,, \quad F_t \Big|_{\rho=0} = \frac{2}{\mathrm{Re}_0}\,, \quad S_t \Big|_{\rho=0} = 0\,. \tag{14}$$

To obtain the conditions for S_t at $\rho = \infty$, we integrate Eq. (11) over ρ from 0 to ∞:

$$\int\limits_0^\infty \frac{\partial P_t}{\partial t'}\, d\rho = \frac{d}{dt'} \int\limits_0^\infty P_t\, d\rho = \frac{\partial q_t}{\partial t'} = -\frac{10}{\mathrm{Re}_0} \frac{\partial P_t}{\partial \rho}\Big|_{\rho=0}\,, \tag{15}$$

where by definition

$$q_t = \int\limits_0^\infty P_t\, d\rho\,.$$

The latter expression makes it possible to formulate the boundary condition at $\rho = \infty$:

$$S_t \Big|_{\rho=0} = q_t\,. \tag{16}$$

Thus, to find the sought functions P_t, F_t, S_t, and q_t, we need to solve the system of equations in partial derivatives (11)-(13) with the boundary conditions (14), (16) as well as the ordinary differential equation (15) with the initial condition $q_t|_{t=0} = 1$.

We integrate Eq. (11) over ρ from ρ to ∞. Taking into account that $\partial P_t/\partial t' = (\partial/\partial t')(\partial S_t/\partial \rho) = (\partial/\partial \rho)(\partial S_t/\partial t')$, we obtain

$$\frac{\partial S_t}{\partial t'} = F_t \left(\frac{\partial P_t}{\partial \rho} + 4 \frac{P_t}{\rho} \right) - \frac{n}{\mathrm{Re}_0}\, G \frac{P_t}{\rho} + \frac{dq_t}{dt'}\,. \tag{17}$$

We transform the system of equations (17), (12), (13), and (15) by introducing the normalized dependent variables $P = P_t|_{q_t}$, $S = S_t|_{q_t}$, $F = F_t\mathrm{Re}_0$, the time $t = t'/\mathrm{Re}_0$, and the dimensionless parameter of the dissipation rate $f_0 = 10(\partial P/\partial \rho)|_{\rho=0} = 2/3 \cdot \mathrm{Re}_0(\varepsilon/q_t)$. We obtain the following system with the boundary and initial conditions:

$$\frac{\partial S}{\partial t} = F \left(\frac{\partial P}{\partial \rho} + \frac{4P}{\rho} \right) - nG\frac{P}{\rho} - 10\,(1 - S)\,\frac{\partial P}{\partial \rho}\Big|_{\rho=0}\,,$$

$$\frac{\partial F}{\partial \rho} = 2\gamma \sqrt{q_t}\, \mathrm{Re}_0 \sqrt{\rho P}\,, \quad \frac{\partial S}{\partial \rho} = P\,, \quad \frac{dq_t}{dt} = -f_0 q_t\,, \tag{18}$$

$$P|_{\rho=0}=0\,,\quad F|_{\rho=0}=2\,,\quad S|_{\rho=0}=0\,,\quad S|_{\rho=\infty}=1\,,\quad q|_{t=0}=1\,.$$

To transform the system of equations (18) to a form convenient for calculations, we introduce, instead of the probability density function P, the related function

$$\varphi=\frac{P}{\rho\left.\dfrac{\partial P}{\partial\rho}\right|_{\rho=0}}\,, \tag{19}$$

as well as the "energy" time $\tilde{t}$, which is liked by a differential relationship to the time t':

$$f_0 dt'=d\tilde{t}. \tag{20}$$

This variable is referred to as the "energy" time since the turbulence energy decays exponentially with this time.

We write the system of equations finally as

$$q_t=\exp\left(-\tilde{t}\right), \tag{21}$$

$$F\left(\rho\frac{\partial\varphi}{\partial\rho}+5\varphi\right)-nG\varphi-10\left(1-S\right)-10\frac{\partial S}{\partial\tilde{t}}=0\,, \tag{22}$$

$$\frac{\partial F}{\partial\rho}=2\gamma\sqrt{q_t}\,\mathrm{Re}_0\,\rho\sqrt{\varphi}\,, \tag{23}$$

$$\frac{\partial S}{\partial\rho}=P\,, \tag{24}$$

$$\frac{\partial f_0}{\partial\rho}=0\,, \tag{25}$$

$$\varphi|_{\rho=0}=1\,,\quad F|_{\rho=0}=2\,,\quad S|_{\rho=0}=0\,,\quad S|_{\rho=\infty}=1\,,\quad q|_{t=0}=1\,. \tag{26}$$

The general scheme for a numerical solution to (21)-(26) consists in calculating q_t by formula (21) for each new time step $\Delta\tilde{t}$ and then solving the nonlinear boundary-value problem for ρ (22)-(26) for the known q_t on in the new step with replacement of the time derivative by its finite-difference approximation $\partial S/\partial\tilde{t}\cong(S-\hat{S})/\Delta\tilde{t}$, where $\hat{S}(\rho)$ is the value of S in the previous step in time $\tilde{t}$. The transition from the energy time to the real time t occurs in the subsequent integration of Eq. (20).

The initial distribution P over the spatial scale was prescribed as

$$P|_{t=0}=2\rho\exp\left(-\rho^2\right). \tag{27}$$

For the functions $\varphi(0,\rho)$, $F(0,\rho)$, and $S(0,\rho)$ we obtain respectively $\varphi(0,\rho)=\exp\left(-\rho^2\right)$, $S(0,\rho)=1-\exp(-\rho^2)$, $F(0,\rho)=2+2\sqrt{2\gamma}\mathrm{Re}_0\left(1-\sqrt{1-S}\right)$. It is easy to calculate the initial value for the constant f_0: $f_0=20$.

The nonlinear boundary-value problem (22)-(26) was solved numerically by the iterative process of solving successive linear two-point boundary-value problems. We represent

$$\varphi=\varphi^*+\delta\varphi\,,\quad F=F^*+\delta F\,,\quad S=S^*+\delta S\,,\quad f_0=f_0^*+\delta f_0$$

and linearize Eqs. (22)-(24) according to Newton. In linearizing we take into account that the function that describes the influence of polymer additives G depends on $\partial P/\partial\rho$ and $\partial P/\partial\rho|_{\rho=0}$. Denoting $y=$

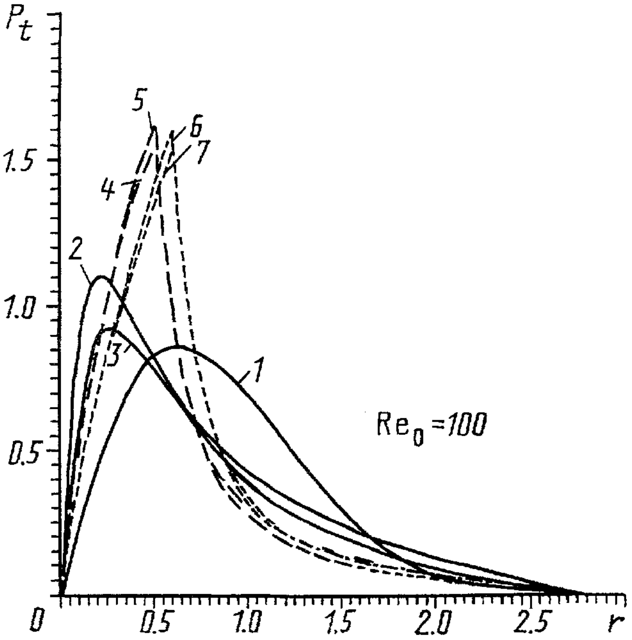

Fig. 1. Shape and evolution of the function $P_t(r)$ vs concentration of polymer additives at $Re_0 = 100$: 1) $n = 0$, $t = 0$; 2) 0 and 1; 3) 0 and 2; 4) 10 and 1; 5) 10 and 2; 6) 100 and 1; 7) 100 and 2.

$(\partial P/\partial \rho)/(\partial P/\partial \rho|_{\rho=0})$, we write $G = G^* + \delta G = G^* + (\partial G/\partial y)\delta y + (\partial G/\partial f_0)\delta f_0$. The linearization procedure is performed in a standard manner, and therefore we do not derive or present the linearized equations. The algebraic boundary-value problem for the corrections $\delta\varphi$, δF, δS, and δf_0 is solved by the two-point matrix run method, described in [5]. It is convenient to assess the convergence of the iteration in the nonlinearity by the value of dissipative losses. The iterations came to a close when $\Delta f_0/f_0 < 0.0005$.

We proceed to a description of the results of a numerical calculation.

3. Results of the Numerical Solution of the Equation for $P_t(\rho)$. Figure 1 shows the shape of the function $P_t(\rho)$ for different values of the concentration n and different values of t for $Re_0 = 100$. By comparing curves 2, 4, and 6 it can be seen that the reaction of the distribution $P_t(\rho)$ to the appearance of the additives consists in a significant change in the shape of this function. The distribution becomes sharper in the region of small length scales. This behavior of the function $P_t(\rho)$ is due to the fact that polymer additives primarily decrease the intensity of the smallest-scale pulsations, which relate to the dissipative spectral region. In this case the energy transfer for larger length scales has not yet deviated from the case of no additives, and therefore there is blocking of the spectrum with the energy of pulsations with larger length scales than the dissipative ones. This leads to a distinct sharpening of the shape of the function $P_t(\rho)$. A further increase in the polymer additive concentration decreases the intensity of pulsations with large length scales, which causes a decrease in turbulent transfer over the entire spectrum, which results in a shift of the distribution $P_t(\rho)$ in the region of large length scales. If we managed to confirm experimentally the above character of the shape of $P_t(\rho)$ as a function of the concentration this would be direct evidence for the adequacy of the proposed theory and the physical mechanism of the interaction of polymer additives and turbulence.

The difference in the character of evolution of the function $P_t(\rho)$ in the absence of small polymer additions and for different values of concentration is demonstrated by comparing the groups of curves (1, 2, 3), (4, 5), and (6, 7), which correspond to different n values. It can be seen that the process of splitting of turbulent vortices in a liquid with polymer additives slows down markedly compared to the case of no additives. A comparison of curves 2, 3 and 5, 6 shows that in the time interval from $t = 1$ to $t = 2$ the reduction in size of the length scales for turbulent velocity pulsations persists in the flow with polymer additives, which characterizes the initial nonequilibrium period of development of the spectrum. At the same time in the absence of polymer additives this process is already completed and there is a slow evolution of the equilibrium spectrum.

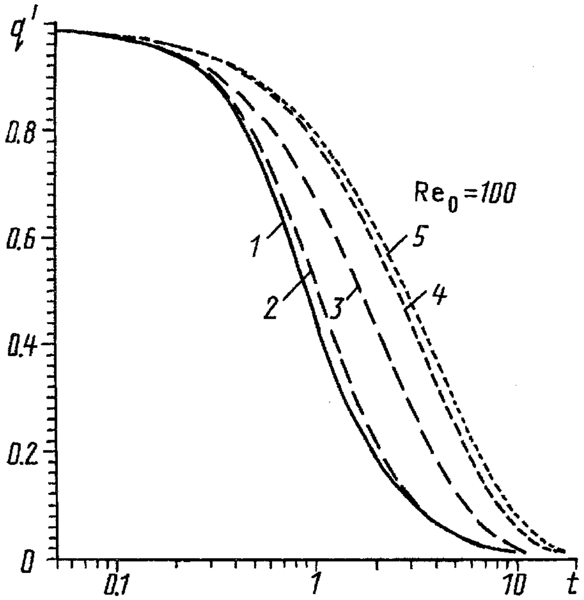

Fig. 2. Turbulent energy evolution for different values of the concentration n and different values of the polymer relaxation time τ: 1) $n = 0$, $\tau = 10^{-3}$; 2) 10 and 10^{-3}; 3) 100 and 10^{-3}; 4) 10 and $2 \cdot 10^{-3}$; 5) 10 and $5 \cdot 10^{-3}$.

TABLE 1. Evolution of the Function $P_t'(0)$ in Time

T	n						
	0	1	10^1	10^2	10^3	10^4	10^5
0.2	8.19	7.0	5.34	4.16	3.55	3.19	3.31
0.6	11.0	9.04	5.84	4.37	3.67	3.29	3.47
1.0	9.72	8.88	5.78	4.32	3.63	3.26	3.46
2.0	5.51	6.36	5.20	4.03	3.43	3.09	3.30
3.0	3.30	3.38	4.31	3.61	3.13	2.85	3.05
4.0	2.21	2.20	3.04	3.00	2.72	2.52	2.67

TABLE 2. Dissipation Rate vs Re_0 and n

n	Re_0	t					
		0	1	2	3	4	5
0	100	2	10.6	6.77	4.87	3.89	2.35
	200	2	20.5	13.5	9.55	7.53	4.40
	800	2	72.5	56.5	38.9	30.0	16,6
1	100	2	9.09	7.76	5.70	4.14	2.31
	200	2	9.05	10.4	10.1	9.64	6.81
	800	2	25.1	51.3	47.5	42.1	18.5
100	100	2	4.33	4.35	4.26	4.14	3.49
	200	2	3.23	3.41	3.45	3.45	3.40
	800	2	2.93	3.09	3.11	3.11	3.09

We can see the effect of small polymer additions on the turbulence energy by comparing the evolution of turbulent energy for different polymer concentration values (Fig. 2).

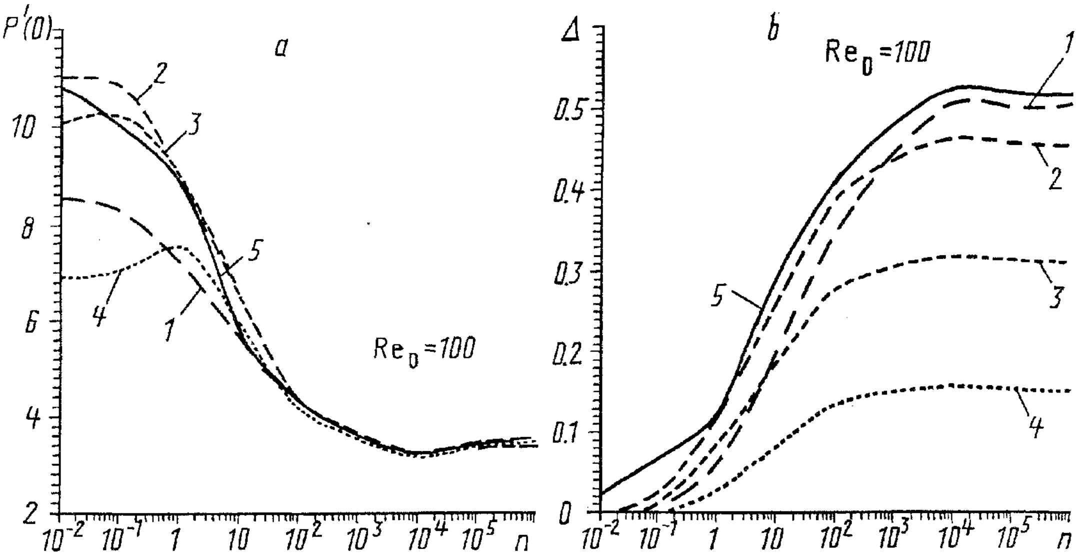

Fig. 3. Dissipation rate (a) and polymer additive efficiency (b) vs concentration fot different values of the turbulent flow energy: 1) $q' = 0.8$; 2) 0.6; 3) 0.4; 4) 0.2; 5) $d = d_{\max}$.

It can be seen from Fig. 2 that in the presence of additives the process of energy decay drags on distinctly prolonged. For example, 80% of the initial energy level for $\mathrm{Re}_0 = 100$ dissipates in $t = 2$ at $n = 0$ and in $t = 3$ at $n = 100$.

The effect of a decrease in the dissipation of the flow is clearly demonstrated by time dependences of the function $P_t'(0) = \lim\limits_{\rho \to 0} (\partial / \partial \rho) P_t(\rho)$, related to the dissipation rate by formula (8), that are calculated for different values of the polymer concentration. It can be seen from Table 1 that the maximum values of dissipation rate may differ by nearly a factor of two for the cases $n = 0$ and $n = 10$. We call attention to the fact that at very large values of polymer concentration the dissipation rate begins to increase ($P'(0)$ is larger at $n = 10^5$ than at $n = 10^4$).

The results of calculating the evolution of the dissipation rate for various Re_0(see Table 2) indicate that polymer additives decrease the maximum rate of energy dissipation by a factor of 1.4–2 and shift the position of the dissipation rate maximum to the right by $\Delta t = 1.2$. The latter circumstance points once again to the slowing down of mixing in a turbulent flow with additives.

Figure 3a shows $P'(0)$ as a function of polymer concentration for different energy levels. It can be seen that for certain values of n $P'(0)$ decreases with increasing n in all cases; however, at large values of n we observe an increase in it.

The effect of a decrease in dissipation is demonstrated most clearly in studying the efficiency of polymer additives. The efficiency is determined as the ratio of the difference of the energy dissipated in the flow with additives and with no additives to the energy dissipated in the flow with no additives:

$$\Delta(n,\ t) = \frac{[1 - q(t,\ 0)] - [1 - q(t,\ n)]}{[1 - q(t,\ 0)]} = \frac{q(t,\ n) - q(t,\ 0)}{1 - q(t,\ 0)}. \tag{28}$$

A calculation (see Table 3) of the evolution of efficiency at different Re_0 shows that the efficiency increases as the flow decreases in scale and then drops to zero. The maximum value of Δ (at approximately $\tilde{t} = 0.1$) attains a value of 0.9 at $n = 100$. Noticeably high efficiency appears at $\mathrm{Re}_0 \sim 75$.

The results of calculating $\Delta(\mathrm{Re}_0)$ (Table 4) show that with increasing Re_0 the efficiency first increases and at large Re_0 values it begins to decrease, which is associated with the "viscous" origin of the effect.

The evolution of efficiency as a function of the polymer additive concentration (Table 5) indicates that the efficiency increases as the concentration increases; however, at very large values of n we observe a decrease in it. The above effect of decreasing efficiency with an increase in polymer additive concentration is also observed experimentally [6]. It is traditionally explained merely by an increase in the ordinary viscosity of the solution.

TABLE 3. Dependence of the Efficiency $\Delta(t)$ for Different n and Re_0

n	Re_0	$\tilde{t}$						
		0.1	0.2	0.4	0.6	1.0	2.0	3.0
1	100	0.002	0.047	0.115	0.115	0.080	0.007	0
	200	0.380	0.480	0.430	0.357	0.240	0.067	0.013
	800	0.641	0.561	0.423	0.322	0.188	0.098	0.001
100	100	0.117	0.331	0.413	0.368	0.257	0.087	0.025
	200	0.676	0.706	0.604	0.500	0.338	0.119	0.027
	800	0.862	0.805	0.662	0.540	0.356	0.119	0.032

TABLE 4. Dependence of the Efficiency $\Delta(\mathrm{Re}_0)$ for Different n

n	Re_0				
	100	200	300	600	1000
1	0.06	0.48	0.57	0.56	0.51
100	0.35	0.70	0.75	0.79	0.78

TABLE 5. Evolution of the Efficiency $\Delta(\tilde{t}, n)$ for Different Concentrations

T	n				
	1	10^1	10^3	10^4	10^5
0.4	0.03	0.11	0.29	0.34	0.35
0.8	0.12	0.29	0.47	0.52	0.51
1.0	0.11	0.29	0.48	0.52	0.51
1.2	0.09	0.28	0.47	0.51	0.50
1.4	0.07	0.26	0.45	0.49	0.48
1.8	0.04	0.21	0.40	0.44	0.43
2.5	0.01	0.13	0.32	0.36	0.34
5.0	0	0.02	0.13	0.16	0.14

This model takes no account of the increase in the ordinary viscosity of the solution with an increase in concentration. The effect of decreasing efficiency at large values of concentration of polymer additives is apparently associated with restructuring of the pulsation spectrum.

Figure 3b shows Δ as a function of n. Different curves correspond to different fixed levels of turbulent energy. The figure illustrates the saturating character of the effect of polymer additives on turbulence. At large values of concentration Δ values do not increase. At very large values of n we observe a decrease in efficiency [6]. The solid curve corresponds to the dependence $\Delta(n)$ at energy values, at which maximum values of efficiency are realized.

Figure 4 shows the evolution of the Reynolds number $\mathrm{Re}(t)$, the time macroscale $T(t)$, and the length macroscale $L(t)$.

The evolution of $\mathrm{Re}(t)$, determined by the formula

$$\mathrm{Re}(t) = \frac{L(t)\sqrt{\left(\frac{2}{3}q(t)\right)}}{\nu},$$

shows that the polymer additives slow down the decrease in the number $\mathrm{Re}(t)$. This is associated with slowing down of the transfer process under the influence of polymer additives.

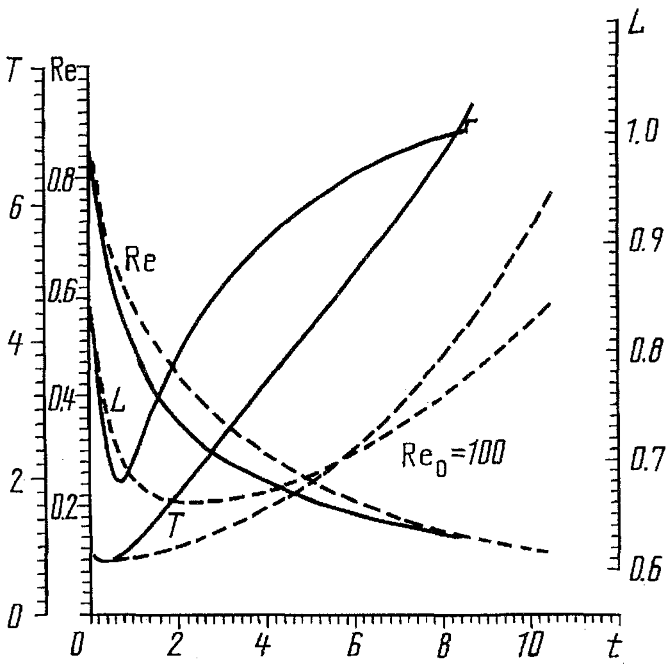

Fig. 4. Influence of polymer additives on the evolution of the Reynolds number Re(t), the time macroscale $T(t)$, and the length macroscale $L(t)$; solid curves) $n = 0$; dashed curves) $n = 100$.

The evolution of the time macroscale

$$T(t) = \frac{L(t)}{\sqrt{\left(\frac{2}{3} q(t)\right)}}$$

demonstrates a reduction in this value by polymer additives at each instant, though the general character of the dependence (the increase in $T(t)$ with time) persists at all values of the concentration n. As a consequence of the decreased rate of turbulent energy dissipation under the influence of polymer additives the rate of growth of the time macroscale slows down.

The evolution of the length macroscale, determined by the formula

$$L(t) = \frac{1}{L_0} \int_0^\infty r P_t(r)\, dr ,$$

shows that the process of turbulent mixing slows down in the presence of polymer additives. The nonequilibrium portion (i.e., the portion where the macroscale decreases) is prolonged. The shape of the function $P_t(r)$ is such that with a strong increase in the microscale and the corresponding decrease in the dissipation rate with increasing n the macroscale $L(t)$, determined by large vortices, decreases. This is a "macroscopic" manifestation of the aforementioned effect of sharpening of the function $P_t(r)$ under the influence of polymer additives.

An interesting and important problem is the study of the rate of mixing as a function of τ which can be changed both by the changing the type of polymer molecules and by changing the initial length scale of the turbulent velocity field (see formula (7) for τ). Figure 2 shows the evolution of turbulent energy for $n = 10$, $\mathrm{Re}_0 = 100$, and for different values of τ. It can be seen from the figure that as τ increases, the process of turbulent energy decay slows down sharply. We can use this rate of turbulent energy decay as a function of τ to control the rate of mixing.

Calculation of the efficiency as a function of τ for different values of Re_0 leads to the conclusion that the situation when τ is large is the most efficient.

A study of the evolution of efficiency as a function of τ shows that as τ increases at a fixed Re, saturation arises, depending on $\Delta(\tau)$. With an increase in Re the maximum efficiency may attain values of about 0.85.

Conclusions. We solved numerically the nonstationary form of the equation, obtained in [1], for the function $P_t(r)$, which describes the evolution in time of the distribution of velocity pulsation energy in an isotropic turbulent flow of dilute linear high-molecular-weight polymers. Results of the numerical solution show that additives lead to a decreased slope of the function $P_t(r)$ at small values of r and to an increased maximum value of this function. The shape of the function changes so that the microscale of turbulent pulsations increases and the macroscale decreases, i.e., we observe the effect of sharpening of the distribution $P_t(r)$. The decreased slope of $P_t(r)$ at small values of r points to decreased dissipation of turbulence under the influence of the additives. A shift of the distribution maximum to the region of large length scales indicates slowed processes of turbulent transfer in a liquid flow with the additives.

An investigation of the evolution of turbulent pulsation energy as a function of the polymer additive concentration shows that half the turbulent energy dissipates more slowly by a factor of 2.5 and the 2% energy level with respect to the initial one is attained in a time 100 times greater in a flow with polymer additives than in a flow with no additives. The maximum value of the dissipation rate is smaller by a factor of 2 in a flow with additives than in a flow with no additives.

It is shown that noticeably high values of the polymer additive efficiency begin to be manifested at initial Reynolds numbers of about 75 and maximum values reach 0.9. We also note an important feature of the efficiency as a function of the Reynolds number. It is found that with an increase in the Reynolds number the efficiency first increases and then begins to decrease. This feature is confirmed experimentally [2, 6] and, as has been shown in [4], is associated with the "viscous" origin of the polymer term in Eq. (1) for $P_t(r)$. This dependence $\Delta(\mathrm{Re})$ is the most direct evidence for the validity of the mechanism of interaction of turbulence with polymer additives, used as the basis of the theory.

An analysis of the results of a numerical calculation of the efficiency Δ for different values of the polymer additive concentration n shows that the usual tendency of increase in Δ with increase in n at large n values gives way to the opposite one. This result is not associated with increased ordinary viscosity since the ordinary viscosity as a function of polymer additive concentration is not taken into account in this model, and it is more likely a consequence of the complex interaction of turbulence and polymer additives, which, at large concentrations, leads to increased turbulent transfer over the spectrum of length scales. This effect, known from experiment, has no theoretically substantiated explanation, as noted in [6].

It is shown that by varying the value of τ (and this variation can be provided both by choosing the type of polymer with different values of relaxation time and by varying the length scale of turbulence) we can control the rate of turbulent mixing and the efficiency regime.

NOTATION

$P_t(r)$, function that describes the distribution of turbulent pulsation energy over different length scales, Re_0, initial Reynolds number; L_0, initial length macroscale of turbulent pulsations; T, characteristic relaxation time for a turbulent molecule; τ, dimensionless relaxation time for polymer molecules; $\varepsilon(t)$, rate of turbulent energy dissipation; $q(t)$, ratio of the energy of turbulent velocity pulsations to the initial value; $\Delta(n, t)$, efficiency of polymer additives; $n = \nu_s/\nu$; ν_s, solution viscosity coefficient at points where molecular tension is realized; ν, solvent viscosity coefficient; $\tilde{t}$, energy time.

REFERENCES

1. V. A. Sosinovich, Inzh.-Fiz. Zh., **61**, No. 6, 895-904 (1991).
2. J. L. Lumley, Polymer Sc. Macromolecular Reviews, 7, 263-290 (1973).
3. J. L. Lumley, Review Fluid Mech., 387-389 (1981).
4. V. A. Sosinovich and V. A. Tsyganov, Inzh.-Fiz. Zh., **61**, No. 6, 905-914 (1991).
5. V. V. Rusanov, Dokl. Akad. Nauk SSSR, **136**, No. 1, 33-38 (1961).
6. D. B. Hoyt, Teor. Osn. Inzh. Raschetov, **94**, No. 2, 1-31 (1972).